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A.YA. DOVGUN, YU.O. USHENKO, O.V. OLAR

STABILIZATION OF DIFFUSION STOCHASTIC DYNAMIC INFORMATION SYSTEMS WITH ACCOUNT OF EXTERNAL RANDOM DISTURBANCES

Yuriy Fedkovych Chernivtsi National University,
2 Kotsjubynskyi Str. Chernivtsi, Ukraine, e-mail: a.dovgun@chnu.edu.ua

Анотація. Одержано умови оптимальної стабілізації дифузійних стохастичних динамічних систем з зовнішніми збуреннями. Метою даної роботи є знаходження умов стабілізації дифузійних стохастичних динамічних систем із зовнішніми випадковими збуреннями.
Тема дослідження є актуальною, оскільки результати роботи дозволять досліджувати стабілізацію динамічних стохастичних систем випадкової структури, що дає можливість у багатьох випадках стабілізувати нестійку стохастичну систему при виконанні певних умов.
Ключові слова: абсолютна стійкість, інформаційна система, автоматичне регулювання, стохастичне рівняння.

Abstract. Conditions for optimal stabilization of diffusive stochastic dynamic systems with external disturbances have been obtained. The purpose of this work is to find the stabilization conditions of diffusive stochastic dynamic systems with external random disturbances.
The research topic is relevant because the results of the work will allow to investigate the stabilization of dynamic stochastic systems of a random structure, which makes it possible in many cases to stabilize an unstable stochastic system when certain conditions are met.
KeyWords: absolute stability, information system, automatic regulation, stochastic equation.

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INTRODUCTION

One of the main conditions for the physical implementation of processes is their stability. The foundations of the theory of stability and control of systems described by stochastic differential equations were laid by M.M. Krasovskyi , I.Ya. Katsom, R.Z. Khasminsky in the early 60s of the 20th century.

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1. STATEMENT OF THE PROBLEM

Let $(\Omega_1, \mathscr{F}_1, \mathbb{P})$ random variables be set on the probability space, and a $f_1(\xi_1(\omega)) \in \mathbb{R}^1 (i = 1,2)$ diffusive stochastic dynamic system with external disturbances (DSDSzZZ) be set on the probabilistic basis $(\Omega_2, \mathscr{F}_2, \mathscr{F}_{2t}(t \geq 0), \mathbb{P})$:

$$dx(t,\omega) = f_1(\xi_1(\omega))a(t,x(t),y(t)),u(t,x(t),y(t)))dt + f_2(\xi_2(\omega))b(t,x(t),y(t))dw(t,\omega) \qquad (1)$$

with initial conditions

$$x(t_0,\omega) = x^0 \in \mathbb{R}^n, y(t_0) = y^0 \in \mathbb{Y}, t_0 \geq 0. \tag{2}$$

In equation (1), a purely discontinuous Markov process $y(t) = y(t,\omega) \in \mathbb{R}^n$allows the decomposition [1]:

$$\mathbb{P}\{y(t+\Delta t) \in (\beta, \beta + \Delta\beta)/y(t) = \alpha \neq \beta\} = p(t,\alpha,\beta)\Delta\beta\Delta t + o(\Delta t), \tag{3}$$

$$\mathbb{P}\{y(\tau) \equiv \alpha, t < \tau \leq t + \Delta t | y(t) = \alpha\} = 1 - p(t,\alpha)\Delta\beta\Delta t + o(\Delta t),$$
$$\alpha, \beta \in \mathbb{Y} = [\eta_1, \eta_2], \tag{4}$$

where $\mathbb{P}(A|B)$ is the conditional probability of the event A , $o(\Delta t)$is an infinitesimally small value of a higher order. Functions $p(t,\alpha,\beta)$; $p(t,\alpha)$- famous.

DSDSzZZ (1), (2) should be considered on a more complex probabilistic basis [1]:

$$(\Omega, \mathscr{F}, \{\mathscr{F}_t, t \geq t_0 \geq 0\}, \mathbb{P}), \tag{5}$$

where $\Omega \equiv \Omega_1 \cap \Omega_2$, $\mathscr{F} \equiv \mathscr{F}_1 \cap \mathscr{F}_2$, $\mathscr{F}_t = \mathscr{F}_{1t} \cap \mathscr{F}_{2t}$, $a(\cdot,\cdot,\cdot): (-\infty, 0) \times \mathbb{D} \times \mathbb{U} \to \mathbb{R}^n$ _$b(\cdot,\cdot,\cdot): 0, \infty) \times \mathbb{D} \times \times \mathbb{U} \to \mathbb{R}^n \times \mathbb{R}^n$, $f_i(\cdot)$are Berel functions, $i = 1,2$; $\xi_i(\omega)$are given random variables, $i = 1,2$, with known distribution laws

$$F_{\xi_i}(x) \equiv \mathbb{P}(\omega: \xi_i(\omega) < x),\ i = 1,2; \tag{6}$$

$\mathbb{D}$– the space of functions that do not have discontinuities of the second kind; $\mathbb{U}$– 2-dimensional control space; $w(t) = w(t,\omega)$– n -dimensional Wiener process.

Since undisturbed motion is studied in (1) $x \equiv 0$, it is natural to put

$$a(t,0,y,0) \equiv 0; b(t,0,y) \equiv 0, \tag{7}$$

and the functions $a(t,x,y,u)$, $b(t,x,y)$are continuous together with their partial derivatives in the domain

$$t \geq t_0 \geq 0,\ x \equiv x(t,\omega) \in \mathbb{R}^n,\ y \in \mathbb{Y}. \tag{8}$$

The control $u\big(t,x,y,f_i(\xi_i)\big)$, $i = 1,2$, is built according to the principle of full inverse communication, i.e. at each moment of time it $t \geq t_0 \geq 0$is possible to accurately measure not only the trajectory of the system (1), (2) $x \equiv x(t,\omega) \in \mathbb{R}^n$, but also the measurement$f_i(\xi_i)$ $i = 1,2$, and the random structure $y(t) \equiv y(t,\omega)$in which the system (1) is located at the given moment of time $t \geq t_0 \geq 0$.

The single control $u\big(t,x[t],y(t),f_i(\xi_i)\big)$, $i = 1,2$, should be chosen under the condition of minimizing the functional

$$J_u(t_0,x^0,y^0) \equiv \int_{t_0}^{\infty} \mathbb{E}\{\mathbb{F}(t,x[t],y(t),u[t])/x(t_0) = x^0,$$
$$y(t_0) = y^0, f_i(\xi_i), i = 1,2\}dt \tag{9}$$

where $\mathbb{F}(t,x,y,u)$ is an integral function defined in domain (8), $u \in \mathbb{R}^2$, is $\mathbb{E}\{\cdot/\cdot\}$ a conditional mathematical expectation [2] ; $u[t] = u\big(t,x(t),y(t),f_i(\xi_i)\big)$is random control implemented by system (1) for $u = u(t,x,y)$, $f_i\big(\xi_i(\omega)\big)$are given random variables with law (6).

The task is to calculate the value of the functional $\mathbb{F}$ (9) with the given control $u \equiv u\big(t,x,y,f_i(\xi_i)\big)$, $i = 1,2$, which means that it is necessary to find the trajectories of the $x[t]$system (1), (2), and then substitute $x[t]$, $y(t)$, $u[t] \equiv u\big(t,x[t],y(t)\big)$, $f_i\big(\xi_i(\cdot)\big)$, $i = 1,2$into the functional (9) and limit its value [2], [4].

2. STUDY OF STABILIZATION CONDITIONS OF DIFFUSIVE STOCHASTIC DYNAMIC SYSTEMS WITH EXTERNAL RANDOM DISTURBANCES

The question of choosing a functional $\mathbb{F}(t,x,u)$that determines the assessment $J_u$of process quality $x[t]$depends on the specific features of the task. But, as a rule, the main role is played by the following conditions:

I) the condition of the minimum of the functional (9) must ensure a sufficiently fast, on average, decay of the process $x[t]$;

II) the value of the integral$J_u\left(t_0,x,f_i(\xi_i)\right)$ must satisfactorily evaluate the energy resources used for the formation of control;

III) the functional $\mathbb{F}\left(t,x,u,f_i(\xi_i)\right)$, $i=1{,}2$, should be such that the solution of problem (1), (2), (9) can be obtained in a constructive form.

For a linear system of type (1), (2), the functional $\mathbb{F}$should be chosen in the form of a quadrature form [2] :

$$\mathbb{F}\left(t,x,u,f_i(\xi_i)\right)=x^TC(t)x+u^TL(t)u,\ i=1{,}2, \tag{10}$$

where $C(t)$is a symmetric nonnegative $n\times n$ -matrix, $L(t)$is a positive definite $n\times n$ -matrix for $\forall t\geq t_0\geq 0$.

Based on the above, the following problem of optimal stabilization is considered, which we will formulate in the form of a statement.

Statement 1.

Let

1) on a probabilistic basis $(\Omega,\mathscr{F},\{\mathscr{F}_t,t\geq t_0\geq 0\},\mathbb{P})$ problem (1), (2), (9) is considered under the condition of jump (3), (4);

2) such a scalar function $v^0\equiv v^0\left(t,x,y,f_i(\cdot)\right)$ and a 2-dimensional function $u^0\equiv u^0\left(t,x,y,f_i(\cdot)\right)$defined in domain (2) and satisfies the conditions:

positive definite for $x$and $f_i(\cdot)$in region (2);

allows an infinitely small upper and an infinitely large lower limit;

the functional $\mathbb{F}$from criterion (9) is positive definite for $x$and $\mathbb{E}\{f_i(\cdot)\}$;

averaged derivative $\left(\frac{d\mathbb{E}[v^0]}{dt}\right)_{u^0}$calculated by the formula:

$$u^0\equiv u^0(t,x,y,\mathbb{E}\{f_i(\cdot)\})$$

and satisfies the condition

$$\left(\frac{d\mathbb{E}[v^0]}{dt}\right)_{u^0}=-\mathbb{F}\left(t,x,y,u^0,f_i(\cdot)\right) \tag{11}$$

value $\left(\frac{d\mathbb{E}[v^0]}{dt}\right)_{u^0}+\mathbb{F}\left(t,x,y,u,f_i(\cdot)\right)$reaches a minimum at $u=u^0$, i.e

$$\left(\frac{d\mathbb{E}[v^0]}{dt}\right)_{u^0}+\mathbb{F}(t,x,y,\mathbb{E}\{f_i(\xi_i)\})=\min_{u\in\mathbb{R}^2}\left\{\frac{d\mathbb{E}[v^0]}{dt}\right\}_{u^0}+\mathbb{F}(t,x,y,u,\mathbb{E}\{f_i(\xi_i)\})=0. \tag{12}$$

Then:

I) control $u^0\left(t,x,y,f_i(\cdot)\right)$performs optimal stabilization;

II) equality is fulfilled

$$v^0\left(t_0,x^0,y^0,\mathbb{E}\{f\left(\xi_i(\omega)\right)\}\right)=\int_{t_0}^{\infty}\mathbb{E}[\mathbb{F}(t_0,x^0[t],y(t),u^0[t])\,/x(t_0)=x^0,y(t_0)=y^0,f_i\left(\xi_i(\cdot)\right)\big]dt=$$
$$=\min\int_{t_0}^{\infty}\mathbb{E}[\mathbb{F}(t,x[t],y(t),u[t])/\,x^0,y^0,f_i(\cdot)]dt=J_{u^0}\left(t_0,x^0,y^0,f_i(o_i)\right). \tag{13}$$

Proof. The asymptotic stability of probability in general DSDSzZZ (1) – (2) under the condition of jump (3) for $u\equiv u^0\left(t,x,y,f_i(\xi_i)\right)$, $i=1{,}2$, immediately follows from the fact that the function $v^0(t,x,y,\mathbb{E}\{f_i(\xi_i)\})$satisfies all the necessary conditions.

Theorem 1. Suppose that for DSDSzZZ (1) – (3), which is given on a probabilistic basis $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$: 1) there is a positive definite function $v(t, x, y, \mathbb{E}\{f_i(\xi_i)\})$that admits an infinitely small upper limit;

2) the averaged derivative $\frac{d\mathbb{E}[v]}{dt} < 0$is negative in the region $x \in \mathbb{R}^n$, $y \in \mathbb{Y}$, $t \geq t_0$; $\mathbb{E}\{f_i^2(\xi_i)\} < K$, $i = 1,2$.

Then undisturbed movement$x = 0$ is asymptotically stable in probability.

The proof of the stabilization of the solution of problem (1) – (3) consists in the assumption of the existence of control $u^*\big(t, x, y, f_i(\xi_i)\big) \neq u^0\big(t, x, y, f_i(\xi_i)\big)$, $i = 1,2$, which is substituted into equation (1) gives such a solution $x^*[t]$, under the initial conditions (2), that the inequality with probability 1 is satisfied.

$$J_{u^*}\big(t_0, x^0, y^0, f_i(\xi_i)\big) < J_{u^0}\big(t_0, x^0, y^0, f_i(\xi_i)\big), \; i = 1,2. \tag{14}$$

Condition (12) gives the inequality

$$\left(\frac{d\mathbb{E}[v^0]}{dt}\right)_{u^*} \geq -\mathbb{F}(t, x, y, u^*(t, x, y)). \tag{15}$$

Next, (15) should be averaged over the coincidences ($x^*[t]$, $y(t)$, $f_i\big(\xi_i(\omega)\big)$, $i = 1,2$), integrate the $t$resulting expression from $t_0$to taking into account the $t = T < \infty$Dynkin formula [2] . As a result, the inequality holds:

$$\begin{gathered}\mathbb{E}\left[v^0\left(T, x^*[T], y[T]/x^0, y^0, f_i\big(\xi_i(\omega)\big)\right)\right] - v^0(t_0, x^0, y^0) \geq \\ \geq -\int_{t_0}^{T} \mathbb{E}\left[\mathbb{F}\left(t, x^*[t], y(t), u^*[t], f_i\big(\xi_i(\omega)\big)\right)/x^0, y^0\right] dt \geq \\ \geq -\int_{t_0}^{\infty} \mathbb{E}\left[\mathbb{F}\left(t, x^*[t], y(t), u^*[t], f_i\big(\xi_i(\omega)\big)\right)/x^0, y^0\right] dt.\end{gathered} \tag{16}$$

From (14) follows the inequality with probability 1:

$$\begin{gathered}v^0\left(t_0, x^0, y^0, f_i\big(\xi_i(\omega)\big)\right) - J_{u^0}\left(t_0, x^0, y^0, f_i\big(\xi_i(\omega)\big)\right) \leq \int_{t_0}^{\infty} \mathbb{E}\big[\mathbb{F}(t, x^*[t], y(t), u^*[t])/x^0, y^0, f_i\big(\xi_i(\omega)\big)\big]dt = \\ = J_{u^*}\left(t_0, x^0, y^0, f_i\big(\xi_i(\omega)\big)\right),\end{gathered} \tag{17}$$

for $T \to \infty$ the integral in the right-hand side of (16) coincides.

Indeed, the convergence of the integral with probability 1 in the right-hand side of inequality (16) implies the convergence to zero of the integral expression (17) at $t \to \infty$. And this gives a probability of 1 statement

$$\lim_{T\to\infty} \mathbb{E}\left[v^0\left(T, x^*[T], y(T), f_i\big(\xi_i(\omega)\big)\right) = 0.\right.$$

Then inequality (17) contradicts assumption (14). This contradiction proves the statement about the optimality of control $u^0\left(t, x, y, f_i\big(\xi_i(\omega)\big)\right)$, $i = 1,2$, with probability 1. The statement 1 is proved.

Remarks 1. In the above proof limited to the natural case, when the condition $\mathbb{E}[\mathbb{F}] \to 0$implies $\mathbb{E}[v^0] \to 0$.

Theorem 2. Let it be on a probabilistic basis $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}, \mathbb{P})$:

given by DSDSzZZ (1), where $x \in \mathbb{R}^n$; $t \geq t_0$, $y(t)$is a simple Markov chain with a finite number of states $\{y_1, \dots, y_n\} = Y$and given by transition probabilities (3), (4);

$$\mathbb{E}\{f_i^2\big(\xi_i(\omega)\big)\} \leq K < +\infty, \; i = 1,2;$$

at the time of the τchange in the system structure, $y_i \to y_j$there is a random jump-like change in the phase vector - solution $x(\tau - 0) = x$, $x(\tau) = z$, for which the conditional density is given$p_{ij}(\tau, z)$

$$\mathbb{P}\{x(\tau) \in [z, z + \Delta z] | x(\tau - 0) = x\} = p_{ij}(\tau, z/2)dz + o(\Delta z); \tag{18}$$

there exist a scalar function $v$and a 2-dimensional function $u$that satisfy the conditions of statement 1. Then the averaged derivative $\frac{d\mathbb{E}[v]}{dt}$is calculated by the formula

$$\frac{d\mathbb{E}[v]}{dt}=\frac{\partial v}{\partial s}+\left(\frac{\partial v}{\partial x}\right)\mathbb{E}\{f_1(\xi_1)\}a(s,x(s),y_i,u)+\frac{1}{2}tz\left(\frac{\partial^2 v}{\partial x^2}\mathbb{E}\{f_2(\xi_2)\}b(s,u(s),y_i)\cdot b^T(s,x(s),y_i)\right)+$$
$$+\sum_{j\neq i}^{k}\left[\left[\int_{\mathbb{R}_+}v(s,z,y_j)p_{ij}(s,z/x)dz-v(s,x,y_i)\right]q_{ij}.\right.$$

Let a controlled system be given on a probabilistic basis

$$dx=\left[f_1(\xi_1(\omega))A(y(t))x+f_2(\xi_2(\omega))B(y(t))u\right]dt+\sum_{j=1}b_l\,y(t)xdw_j \qquad (19)$$

with initial conditions

$$t_0=0,\ x(0)=x^0,\ y(0)=y^0,$$
$$\mathbb{E}\{f_i^2(\xi_i(\omega))\}\leq K<\infty, (20)$$

where $x\in\mathbb{R}^n$, $u\in\mathbb{R}^2$, $A(y)$, $B(y)$, $b_j(y)$are known matrix-functions, which are given on the set $Y\equiv\{y_1,\ldots,y_n\}$ of possible values of a homogeneous Markov chain $y(t)$, $w(t)\equiv w(t,\omega)$is a stationary one-$l$dimensional Wiener process with independent components $w_j(t)$.

Remark 2. The condition of the jump of the phase vector $x(t)$at the moment $t=\tau$changes the structure of the system due to the transition $y(\tau-0)=y_i$to $y(\tau)=y_j\neq y_i$, are linear, i.e

$$x(\tau)=k_{ij}x+\sum_{s=1}^{v}d_s\,Q_sx, \qquad (21)$$

where $x=x(\tau-0)$, $d_i=d_i(\omega)$are independent random variables with $\mathbb{E}d_s=0$, $\mathbb{E}d_s^2=1$, $K_{ij}$are known elements of the dimension matrix $n\times n$.

The quality of the transition process is estimated by the quadratic functional

$$J_u(x^0,y^0)=\int_0^\infty\mathbb{E}\left[x^T[t]C(y(t))x[t]+u^1[t]D(y(t))u[t]\mathbb{E}\{f_i(\xi_i(\omega))\}/x^0,y^0\right]dt,\ i=1{,}2, \qquad (22)$$

where $C(y)\geq 0$, $D(y)>0$are symmetric matrices of dimension $n\times n$and2×2 in accordance.

We look for the optimal Lyapunov function in the form of a quadratic form

$$v^0(x,y)=x^TG(y)x, \qquad (23)$$

where $G(y)>0$is a positive definite symmetric matrix.

CONCLUSIONS

Let the system of matrix quadratic equations

$$\mathbb{E}\{f_1(\xi_1)\}[G_iA_i+A_i^TG_i]-\mathbb{E}\{f_2^2(\xi_2)\}\left[G_iB_jD_i^{-1}B_i^TG_i+\sum_{j=1}^{l}b_{ji}^TG_ib_{ji}\right]+$$
$$+\sum_{j=i}^{l}\mathbb{E}\{f_1(\xi_1)\}\left(K_{ij}^TG_jK_{ij}+\sum_{s=1}^{N}Q_s^TG_jQ_s-G_i\right)q_{ij}+C_i=0,\ i=\overline{1,k} \qquad (24)$$

where $A(y_i)=A_i$, $b_j(y_i)=b_{ji}$etc., has a solution $G_1,\ldots,G_k$.

Then control

$$u_i^0(x)=-D_i^{-1}B_i^TG_ix \qquad (25)$$

gives the optimal stabilization of the system (19) under the jump condition (21) and the optimality criterion (22);

$$\min_u J_u(x^0,y_i)=(x^0)^TG_ix^0. \qquad (26)$$

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DOVGUN ANDRIY - Ph.D., Associate Professor of Computer Science Department, Yuriy Fedkovich Chernivtsi National University, Chernivtsi, Ukraine, ***e-mail: a.dovgun@chnu.edu.ua***

USHENKO YURIY - D.Sc., Professor of Computer Science Department, Yuriy Fedkovich Chernivtsi National University, Chernivtsi, Ukraine, ***e-mail: y.ushenko@chnu.edu.ua***

OLAR OLEKSANDR – Ph.D., assistant professor of Computer Science Department, Yuriy Fedkovich Chernivtsi National University, Chernivtsi, Ukraine, ***e-mail: o.v.olar@chnu.edu.ua***