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# МЕТОДИ ТА СИСТЕМИ ОПТИКО-ЕЛЕКТРОННОЇ І ЦИФРОВОЇ ОБРОБКИ ЗОБРАЖЕНЬ ТА СИГНАЛІВ

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## METHODS AND SOFTWARE FOR CAPTURING THREE-DIMENSIONAL DIGITAL IMAGES USING SPATIAL DEMODULATION

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**Анотація.** У статті запропоновано узагальнений метод демодуляції, який може бути застосований для захоплення цифрових тривимірних зображень в оптичних мікроскопах. Він дозволяє розрахувати контраст гармонічної просторової модуляції з будь-якими значеннями фазових кутів. Вхідними даними є стек цифрових зображень з гармонічною просторовою модуляцією. Три цифрові зображення з різними фазами модуляції, зняті в одній позиції фокусування, обробляються для отримання одного набору даних про розподіл контрасту. Ці дані дозволяють скласти тривимірні зображення шляхом вибору максимальних значень контрасту в тривимірному просторі. У статті наведено аналітичний розв'язок системи з трьома лінійними рівняннями, що дозволяє ідентифікувати основні параметри гармонічної просторової модуляції, включаючи її середнє значення та контраст. В результаті вираз для розрахунку контрасту присутній в аналітичній формі. Цей вираз справедливий для будь-яких фазових кутів модуляції, які не збігаються. Стаття містить моделювання в середовищі MathCAD, яке ілюструє процедуру складання тривимірних зображень. Моделювання підтверджує правильність описаних розрахунків і демонструє побудову тривимірного зображення поверхні як композиції карти розподілу контрасту та відповідної карти висот. Перевагою запропонованої методики є можливість застосування економних вузлів для формування гармонійної просторової модуляції. Ці пристрої можна встановити на більшість оптичних мікроскопів із цифровими камерами та моторизованими приводами фокусування без придбання дорогого комерційного програмного забезпечення. Це значно здешевлює перетворення цих оптичних мікроскопів на інструменти для отримання цифрових тривимірних зображень.

**Ключові слова:** цифрова обробка зображень, тривимірне цифрове зображення, просторова модуляція освітлення, оптичний мікроскоп, демодуляція.

**Abstract.** The paper proposes the generalized technique of demodulation, which can be applied for capturing digital three-dimensional images in optical microscopes. This technique allows the calculation of the contrast of harmonic spatial modulation with any values of phase angles. The input data is a stack of digital images with harmonic spatial modulation. Three digital images with different phases of modulation captured in the same focus position are processed to get one data set about the contrast distribution. This data makes it possible to compose the three-dimensional images by the selection of the maximal values of the contrast in three-dimensional space. The paper contains the analytical solution of the system with three linear equations that helps to identify the principal parameters of harmonic spatial modulation, including its average value and contrast. As a result, the expression for contrast calculation is present in an analytical form. This expression is valid for any modulation phase angles that do not coincide. The paper contains MathCAD modeling, which illustrates the procedure of composing the three-dimensional images.

The modeling confirms the correctness of described calculations and demonstrates the built of the three-dimensional surface image as a composition of the contrast distribution map and the corresponded height map. The advantage of the proposed technique is the possibility to apply the economical units for the formation of a harmonious spatial modulation. These units can be installed on most optical microscopes with digital cameras and motorized focusing drives without purchasing expensive commercial software. It significantly reduces the price of transforming these optical microscopes into instruments for capturing digital three-dimensional images.

**Keywords:** digital image processing, three-dimensional digital image, spatial modulation of illumination, optical microscope, demodulation.

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## INTRODUCTION

One of the important advantages of modern optical microscopy is the ability to obtain three-dimensional images with submicron resolution in various optical ranges. Thus, confocal laser scanning microscopes use the principle of mechanical scanning with a focused laser beam in three-dimensional space, where the sample for observation is located. Such microscopes make it possible to obtain non-contact three-dimensional images of living cells while observing the fluorescence of organic molecules, surfaces of various materials and high-resolution details. Unfortunately, the high cost of such microscopes, which in most cases ranges from 50,000 to 250,000 Euros, significantly limits access to them by Ukrainian researchers. [1-3]

About 25 years ago, a method for obtaining three-dimensional images in optical microscopes using spatial modulation of illumination (SMI) was proposed. It is known that in the object plane of an optical microscope it is possible to project the image structure of a contour that creates a harmonic SMI. As a result, the maximum contrast illumination will be only in those objects of observation that are very close to the object area of the optical system of the microscope. With small deviations of the surface of the object from the plane of the object, the modulation contrast decreases sharply. That is, the modulation contrast with the correct selection of the modulation frequency is very sensitive to defocusing. Therefore, the following possibility of constructing three-dimensional images appears:

- in the optical microscope, a harmonic spatial modulation of illumination is formed in the plane of the object;
- the object of observation moves along the optical axis relative to the plane of the object with a step smaller than the depth of the sharply imaged space;
- a set of digital images from SMI is formed;
- SMI images are processed, areas with maximum SMI contrast are found, and a three-dimensional image of the surface or three-dimensional structure of the observation object is formed from them.[4-9]

The main advantage of this method of obtaining three-dimensional images is the possibility of using ordinary optical microscopes with digital cameras and motorized focusing drives. The SMI is created by a special device that is inserted into the place of the adjustable field diaphragm of the illumination channel of the microscope. Another significant advantage of this method is the ability to obtain three-dimensional images with a resolution that approaches the resolution of a confocal laser microscope, including color images, at a cost much lower than the cost of a confocal laser microscope. Thus, the cost of PMO devices and software for obtaining three-dimensional images produced by the companies Zeiss or Optigrad exceeds 10,000 - 15,000 Euros. This high price is due to the fact that the SMI device must provide a very precise SMI phase shift to obtain SMI phase values, which in most cases are equal to 0, 120 and 240 degrees. [8-12]

The authors believe that the cost of SMI devices can be significantly reduced by using devices for less accurate SMI displacements and using free and open access software. This will significantly reduce the cost of SMI devices and thereby provide a large number of users with the opportunity to receive three-dimensional images. But for this you need to have a mathematical apparatus, preferably in the form of analytical expressions, which allows you to calculate the contrast of the SMI with any values of the phase angles.

The purpose of the work is the development of a mathematical apparatus (method) for demodulation - calculation of the contrast of SMI – with any values of phase angles and the corresponding software for obtaining three-dimensional digital images.

### 1. SOLUTION OF THE DEMODULATION PROBLEM IN TRIGONOMETRIC FORM

One-dimensional SMI provides maximum contrast. Therefore, we will consider the case of a one-dimensional harmonic SMI along the  $x$  axis, which has certain constant components, amplitude, spatial frequency, and initial phase and is described by the function  $\cos()$ . To find the contrast of the SMI, we describe the one-dimensional SMI as a trigonometric function:

$$\begin{aligned} U(x) &= U_0 + \Delta U \cdot \cos(2\pi \cdot v_x \cdot x + \varphi_0) = \\ &= U_0 + (\Delta U \cdot \cos(2\pi \cdot v_x \cdot x)) \cdot \cos(\varphi_0) - \\ &\quad - (\Delta U \cdot \sin(2\pi \cdot v_x \cdot x)) \cdot \sin(\varphi_0) = \\ &= U_0 + a \cdot \cos(\varphi_0) - b \cdot \sin(\varphi_0); \end{aligned} \tag{1}$$

where  $U(x)$  is a function describing a one-dimensional harmonic signal;  $U_0$  – constant or background component;  $\Delta U$  – SMI contrast, which is equal to the difference between the maximum and minimum values of the harmonic SMI;  $v_x$  is the spatial frequency of the SMI;  $x$  – linear coordinate;  $\varphi_0$  – initial phase of SMI;  $a, b$  are components of the harmonic signal:

$$\begin{aligned} a &= \Delta U \cdot \cos(2\pi \cdot v_x \cdot x); \\ b &= \Delta U \cdot \sin(2\pi \cdot v_x \cdot x). \end{aligned}$$

Expression (1) allows you to write down the formula for determining the SMI contrast:

$$\begin{aligned} \Delta U &= \sqrt{a^2 + b^2} = \\ &= \sqrt{\Delta U^2 \cdot \cos^2(2\pi \cdot v_x \cdot x) + \Delta U^2 \cdot \sin^2(2\pi \cdot v_x \cdot x)} = \\ &= \sqrt{\Delta U^2 \cdot [\cos^2(2\pi \cdot v_x \cdot x) + \sin^2(2\pi \cdot v_x \cdot x)]} = \sqrt{\Delta U^2} \end{aligned} \quad (2)$$

But when calculating the SMI contrast (2), one problem arises: one value  $U(x)$  can be read from the digital image, but three values –  $U_0, a$  and  $b$  – are unknown. In order to know them, you need to have three digital images from SMI, which have different phases of SMI. Then it is possible to find the components  $a, b$  for each pixel of a three-dimensional digital image and calculate the SMI contrast. In other words, it is necessary to compile a linear system of three equations, find its roots ( $U_0, a, b$ ) and calculate the contrast  $\Delta U$ . To simplify the mathematical expressions, we will use the phase angles of the SMI relative to the phase angle of the first digital image with the SMI:

$$\begin{cases} U_1(x) = U_0 + a \cdot \cos(\varphi_1) - b \cdot \sin(\varphi_1) \\ U_2(x) = U_0 + a \cdot \cos(\varphi_2) - b \cdot \sin(\varphi_2) \\ U_3(x) = U_0 + a \cdot \cos(\varphi_3) - b \cdot \sin(\varphi_3) \end{cases} \quad (3)$$

where  $U_1(x), U_2(x), U_3(x)$  are the amplitudes of three digital images with SMI, which have SMI phase angles ( $0, \varphi_2, \varphi_3$ ) at point  $x$ , respectively.

The system of three linear equations (3) can be rewritten in matrix form:

$$\begin{vmatrix} U_1(x) \\ U_2(x) \\ U_3(x) \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & \cos(\varphi_2) & -\sin(\varphi_2) \\ 1 & \cos(\varphi_3) & -\sin(\varphi_3) \end{vmatrix} \cdot \begin{vmatrix} U_0 \\ \Delta U \cdot \cos(2\pi \cdot v_x \cdot x) \\ \Delta U \cdot \sin(2\pi \cdot v_x \cdot x) \end{vmatrix} = \begin{vmatrix} U_1 \\ U_2 \\ U_3 \end{vmatrix} = M \cdot \begin{vmatrix} U_0 \\ a \\ b \end{vmatrix} \quad (4)$$

To find the solution, it is necessary to calculate the inverse matrix  $M^{-1}$  (4):

$$\begin{vmatrix} U_0 \\ a \\ b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & \cos(\varphi_2) & -\sin(\varphi_2) \\ 1 & \cos(\varphi_3) & -\sin(\varphi_3) \end{vmatrix}^{-1} \cdot \begin{vmatrix} U_1(x) \\ U_2(x) \\ U_3(x) \end{vmatrix} = M^{-1} \cdot \begin{vmatrix} U_1(x) \\ U_2(x) \\ U_3(x) \end{vmatrix} = M^{-1} \cdot \begin{vmatrix} U_1 \\ U_2 \\ U_3 \end{vmatrix} \quad (5)$$

where  $M^{-1}$  is the inverse matrix that depends on the phase angles  $\varphi_2, \varphi_3$ :

$$M^{-1} = M^{-1}(\varphi_2, \varphi_3) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & \cos(\varphi_2) & -\sin(\varphi_2) \\ 1 & \cos(\varphi_3) & -\sin(\varphi_3) \end{vmatrix}^{-1} = \frac{1}{D} \cdot \begin{vmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{vmatrix}$$

where  $m_{x,y}$  are the coefficients of the inverse matrix  $M^{-1}$ :

$$\begin{aligned}
 m_{1,1} &= \cos(\varphi_3) \cdot \sin(\varphi_2) - \cos(\varphi_2) \cdot \sin(\varphi_3) = & m_{2,1} &= \sin(\varphi_3) - \sin(\varphi_2) & m_{3,3} &= \cos(\varphi_3) - \cos(\varphi_2) \\
 &= \sin(\varphi_2 - \varphi_3) & m_{2,2} &= -\sin(\varphi_3) & m_{3,3} &= 1 - \cos(\varphi_3) \\
 m_{1,2} &= \sin(\varphi_3) & m_{2,3} &= \sin(\varphi_2) & m_{3,3} &= \cos(\varphi_2) - 1 \\
 m_{1,3} &= -\sin(\varphi_2)
 \end{aligned}$$

D is the determinant of the inverse matrix  $M^{-1}$ , which depends on the phase angles  $\varphi_2, \varphi_3$ :

$$\begin{aligned}
 D &= D(\varphi_2, \varphi_3) = \sin(\varphi_3) - \sin(\varphi_2) - \cos(\varphi_2) \cdot \sin(\varphi_3) + \\
 &+ \cos(\varphi_3) \cdot \sin(\varphi_2) = \sin(\varphi_3) - \sin(\varphi_2) + \sin(\varphi_2 - \varphi_3)
 \end{aligned}$$

Attention is drawn to the fact that the determinant of the matrix M will differ from zero only if the conditions  $\varphi_2 \neq 0, \varphi_2 \neq \varphi_3, \varphi_2 \neq \varphi_3$  are fulfilled. [12,13]

Let's perform mathematical transformations of the  $M^{-1}$  matrix (5) and solve the system of linear equations (4) to calculate the SMI contrast:

$$\begin{aligned}
 M^{-1} &= M^{-1}(\varphi_2, \varphi_3) = \\
 &= \frac{\begin{vmatrix} \sin(\varphi_2 - \varphi_3) & \sin(\varphi_3) & -\sin(\varphi_2) \\ \sin(\varphi_3) - \sin(\varphi_2) & -\sin(\varphi_3) & \sin(\varphi_2) \\ \cos(\varphi_3) - \cos(\varphi_2) & 1 - \cos(\varphi_3) & \cos(\varphi_2) - 1 \end{vmatrix}}{\sin(\varphi_3) - \sin(\varphi_2) + \sin(\varphi_2 - \varphi_3)} \\
 U_0 &= \frac{1}{\sin(\varphi_3) - \sin(\varphi_2) + \sin(\varphi_2 - \varphi_3)} \cdot \\
 &\cdot [U_1 \cdot \sin(\varphi_2 - \varphi_3) + U_2 \cdot \sin(\varphi_3) - U_3 \cdot \sin(\varphi_2)] \\
 a &= \frac{1}{\sin(\varphi_3) - \sin(\varphi_2) + \sin(\varphi_2 - \varphi_3)} \cdot \\
 &\cdot \{U_1 \cdot [\sin(\varphi_3) - \sin(\varphi_2)] - U_2 \cdot \sin(\varphi_3) + U_3 \cdot \sin(\varphi_2)\}; \\
 b &= \frac{1}{\sin(\varphi_3) - \sin(\varphi_2) + \sin(\varphi_2 - \varphi_3)} \cdot \\
 &\cdot \{U_1 \cdot [\cos(\varphi_3) - \cos(\varphi_2)] + U_2 \cdot [1 - \cos(\varphi_3)] + \\
 &+ U_3 \cdot [\cos(\varphi_2) - 1]\} \\
 \Delta U &= 2 \cdot \sqrt{a^2 + b^2} = \frac{2}{|M|} \cdot \sqrt{A^2 + B^2}.
 \end{aligned} \tag{6}$$

Further evolution of the formula (6) can be found in appendix A.

Thus, with the help of the mathematical apparatus (1)-(6), the goal of the work has been achieved - an analytical expression for calculating the SMI contrast with any values of the phase angles has been obtained.

The Zeiss company recommends using the following formula for estimating the SMI contrast when using phases  $\varphi_2 = 2\pi/3$  and  $\varphi_3 = 4\pi/3$  [8,11]:

$$\Delta U = \frac{\sqrt{2}}{3} \cdot \sqrt{(U_1 - U_2)^2 + (U_1 - U_3)^2 + (U_2 - U_3)^2} \tag{7}$$

Note that when using phase angles  $\varphi_2 = 120^\circ = 2\pi/3$  and  $\varphi_3 = 240^\circ = 4\pi/3$ , the obtained formula (Appendix B) gives a result proportional to formula (7), which is used in modern Zeiss Apotome devices. This proves the correctness of the solution to the problem of calculating the SMI contrast (1)-(6). [12,13]

## 2. DISCUSSION

Described demodulation method can be used to obtain three-dimensional images. Consider the construction of an optical microscope for obtaining three-dimensional images. This microscope contains one or two illumination channels for observing samples in reflected or (and) transmitted light with an imaging channel in which the optical system forms a magnified image of the sample in the plane of the matrix photoreceptor of a digital camera. To automate the scanning of the sample, the microscope must have a motorized focusing drive with computer control, which ensures the movement of the sample along the optical axis of the imaging channel with errors several times smaller than the depth of the sharp-imaging space of the microlens. To form a harmonic SMI, a computer-controlled device is used, which allows you to form and move the image of a periodic structure in the object plane of the optical system of the imaging channel in such a way as to ensure certain values of the phase angles  $\varphi_2$  and  $\varphi_3$ . In contrast to the known solutions, the proposed demodulation method (1)-(6) allows processing images from SMI with any phase angles  $\varphi_2$  and  $\varphi_3$ , provided that they do not coincide and are not equal to zero. But it is recommended to design the device so that the phase angles  $\varphi_2$  and  $\varphi_3$  are approximately equal to 120 and 240 angular degrees. The absence of the requirement to provide certain phase angles with high accuracy allows to significantly reduce the cost of the SMI forming device. It should be noted that the vast majority of modern optical microscopes are equipped with digital cameras that are connected to computers and can be equipped with economical computer-controlled motorized focusing drives. Therefore, the economical SMI forming device, which is inserted instead of the field diaphragm adjustment unit of the illumination channel, in the presence of the appropriate software, turns an ordinary optical microscope into an analogue of a confocal laser scanning microscope. [8-11, 14-16]

## 3. SIMULATION OF THE DEMODULATION PROCESS

To check the received data, it is enough to carry out the recovery (demodulation) procedure. The object modulation with three different phase shifts proposed by Zeiss ( $\varphi_2 = 2\pi/3$  and  $\varphi_3 = 4\pi/3$ ) and phases with certain error and also further demodulation procedures are shown at figures 1-4.

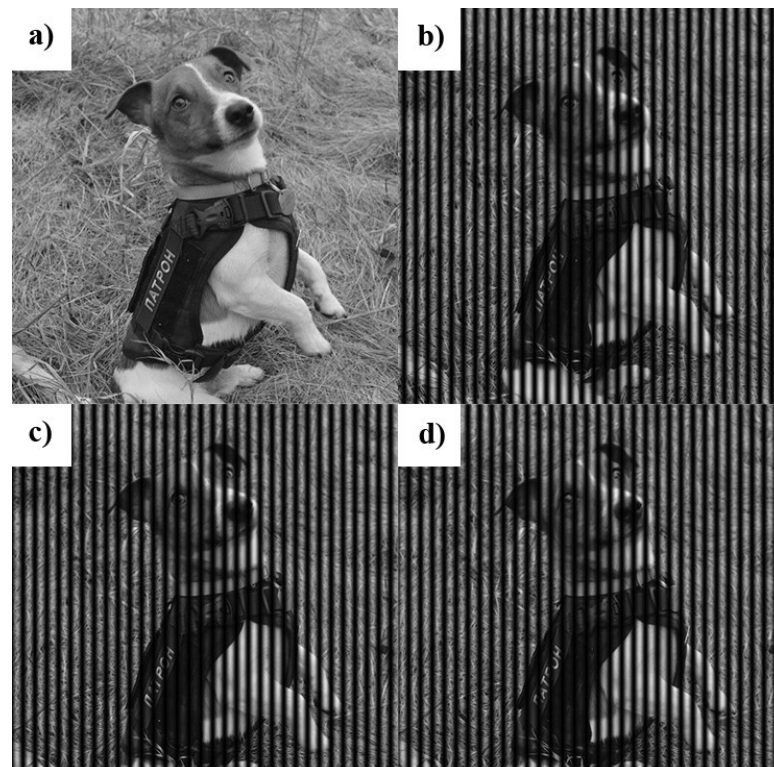


Figure 1 – Images without modulation (a) and with modulation:  
b)  $\varphi_0 = 0$ , c)  $\varphi_0 = 2\pi/3$ , d)  $\varphi_0 = \varphi_0 = 4\pi/3$

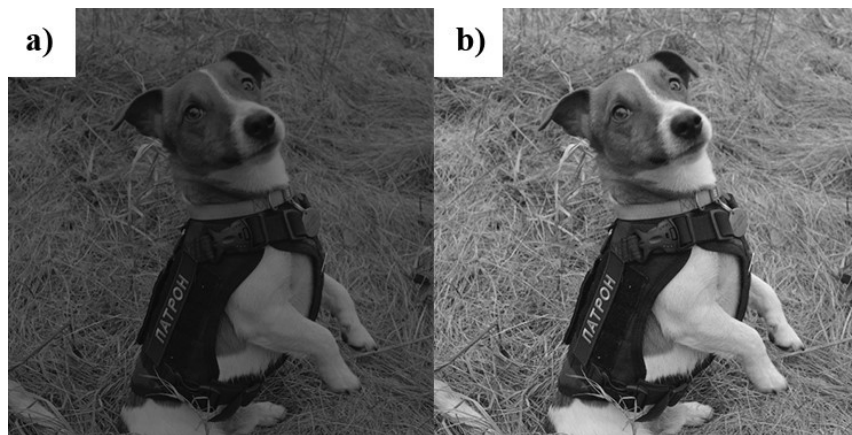


Figure 2 – Demodulation results using the proposed technique,  
modulation phases  $(0, 2\pi/3, 4\pi/3)$ :  
a) output image, b) output image after linear contrast enhancement

So, when setting the problem of demodulation in microscopes with SMI, the input data for the calculation are 3 image signals -  $U_1, U_2, U_3$ , obtained at different phase shifts of the periodic transparent. In real conditions, errors arise due to the noise component.

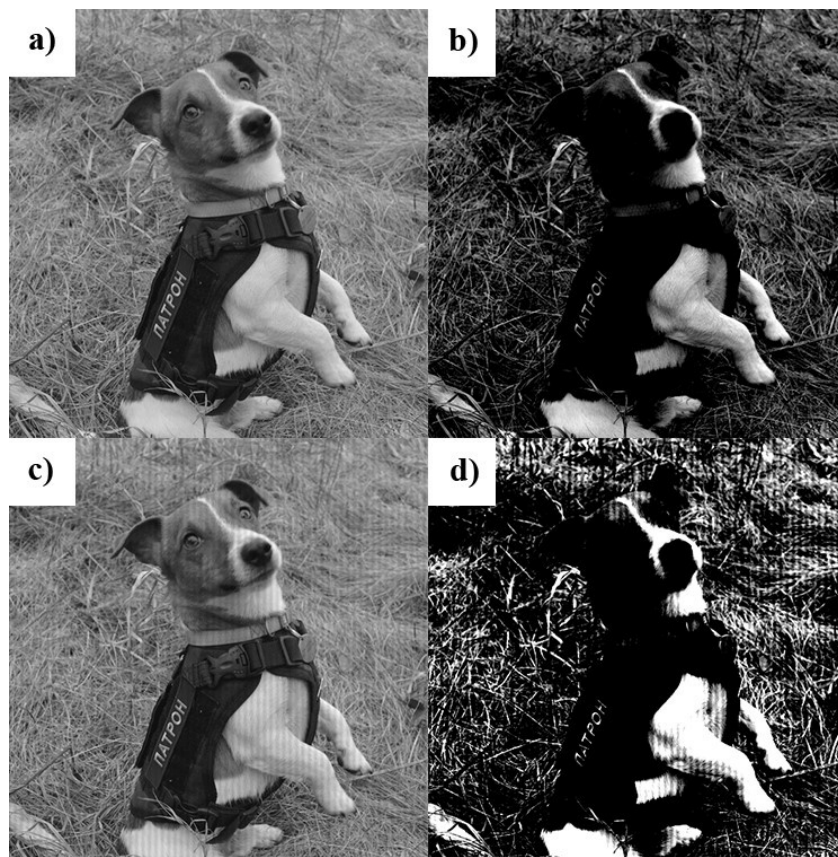


Figure 3 – Demodulation results using the proposed technique,  
modulation phases  $(0, 0.95 \cdot 2\pi/3, 1.05 \cdot 4\pi/3)$ :  
a) proposed technique, b) proposed technique - output image after high contrast enhancement,  
c) known technique, d) known technique - output image after high contrast enhancement,

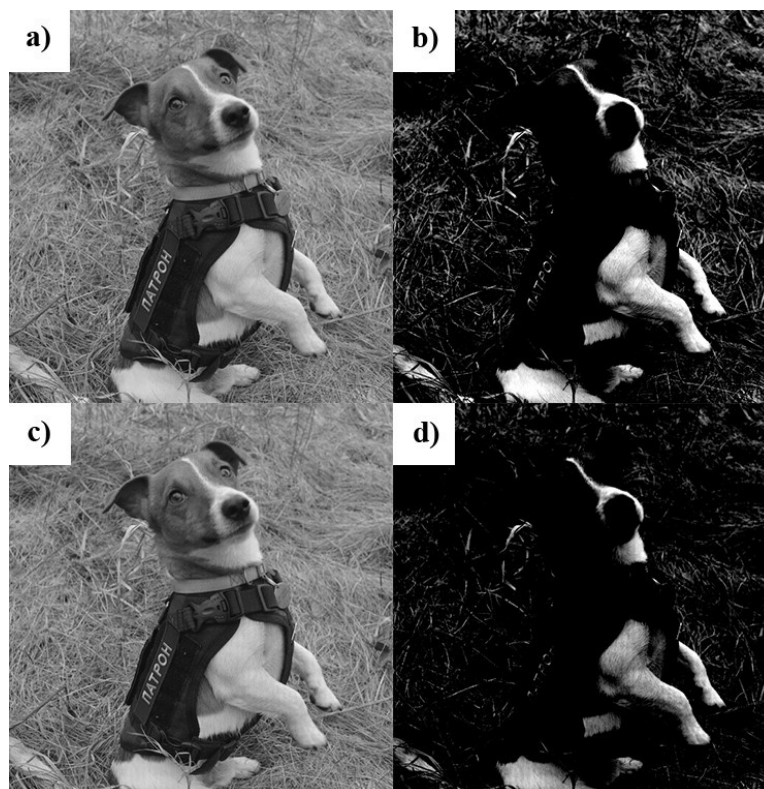


Figure 4 – Demodulation results using the proposed technique,  
modulation phases  $(0, 0.99 \cdot 2\pi/3, 1.01 \cdot 4\pi/3)$ :  
a) proposed technique, b) proposed technique - output image after high contrast enhancement,  
c) known technique, d) known technique - output image after high contrast enhancement,

### CONCLUSIONS

The paper proposes a generalized method of demodulation and an analytical expression for calculating the contrast of harmonic SMI with any values of SMI phase angles. This method can be applied in software for obtaining three-dimensional digital images. The work contains a model based on the proposed method in the MathCAD environment for obtaining three-dimensional images (Appendix C). The main advantage of the proposed method is that it can be used with economical SMI devices on most optical microscopes without purchasing commercial software. A further direction of research may be the choice of the design of economic SMI devices and the study of the process of obtaining three-dimensional digital images and different modes of observing samples.

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**APPENDIX A**

$$\Delta U = 2 \cdot \sqrt{a^2 + b^2} = \frac{2}{|M|} \cdot \sqrt{A^2 + B^2}.$$

$$\begin{aligned} A^2 + B^2 &= \{U_1 \cdot [\sin(\varphi_3) - \sin(\varphi_2)] - U_2 \cdot \sin(\varphi_3) + U_3 \cdot \sin(\varphi_2)\}^2 + \\ &+ \{U_1 \cdot [\cos(\varphi_3) - \cos(\varphi_2)] + U_2 \cdot [1 - \cos(\varphi_3)] + U_3 \cdot [\cos(\varphi_2) - 1]\}^2 = \\ &= U_1^2 \cdot \{[\sin(\varphi_3) - \sin(\varphi_2)]^2 + [\cos(\varphi_3) - \cos(\varphi_2)]^2\} + \\ &+ U_2^2 \cdot \{[-\sin(\varphi_3)]^2 + [1 - \cos(\varphi_3)]^2\} + U_3^2 \cdot \{[\sin(\varphi_2)]^2 + [\cos(\varphi_2) - 1]^2\} + \\ &+ 2 \cdot U_1 \cdot U_2 \cdot \{[\sin(\varphi_3) - \sin(\varphi_2)] \cdot [-\sin(\varphi_3)] + [\cos(\varphi_3) - \cos(\varphi_2)] \cdot [1 - \cos(\varphi_3)]\} + \\ &+ 2 \cdot U_2 \cdot U_3 \cdot \{[-\sin(\varphi_3)] \cdot \sin(\varphi_2) + [1 - \cos(\varphi_3)] \cdot [\cos(\varphi_2) - 1]\} + \\ &+ 2 \cdot U_1 \cdot U_3 \cdot \{[\sin(\varphi_3) - \sin(\varphi_2)] \cdot \sin(\varphi_2) + [\cos(\varphi_3) - \cos(\varphi_2)] \cdot [\cos(\varphi_2) - 1]\} = \\ &= 2 \cdot U_1^2 \cdot [1 - \cos(\varphi_3 - \varphi_2)] + 2 \cdot U_2^2 \cdot [1 - \cos(\varphi_3)] + 2 \cdot U_3^2 \cdot [1 - \cos(\varphi_2)] + \\ &+ 2 \cdot U_1 \cdot U_2 \cdot \{\cos(\varphi_3 - \varphi_2) + \cos(\varphi_3) - \cos(\varphi_2) - 1\} + \\ &+ 2 \cdot U_2 \cdot U_3 \cdot \{-\cos(\varphi_3 - \varphi_2) + \cos(\varphi_3) + \cos(\varphi_2) - 1\} + \\ &+ 2 \cdot U_1 \cdot U_3 \cdot \{\cos(\varphi_3 - \varphi_2) - \cos(\varphi_3) + \cos(\varphi_2) - 1\}. \end{aligned}$$

$$\Delta U = \frac{2 \cdot \sqrt{A^2 + B^2}}{|\sin(\varphi_3) - \sin(\varphi_2) + \sin(\varphi_2 - \varphi_3)|}.$$



**APPENDIX B**

To check the obtained formula (6), the SMI contrast when using phases  $\varphi_2 = 2\pi/3$  and  $\varphi_3 = 4\pi/3$ :

$$\Delta U = \left[ \frac{1}{\sin\left(\frac{4\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3} - \frac{4\pi}{3}\right)} \right]^2 \cdot \left\{ U_1 \cdot \left[ \sin\left(\frac{4\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right) \right] - U_2 \cdot \sin\left(\frac{4\pi}{3}\right) + U_3 \cdot \sin\left(\frac{2\pi}{3}\right) \right\}^2 + \left\{ U_1 \cdot \left[ \cos\left(\frac{4\pi}{3}\right) - \cos\left(\frac{2\pi}{3}\right) \right] + U_2 \cdot \left[ 1 - \cos\left(\frac{4\pi}{3}\right) \right] + U_3 \cdot \left[ \cos\left(\frac{2\pi}{3}\right) - 1 \right] \right\} \cdot \left[ \frac{1}{\sin\left(\frac{4\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3} - \frac{4\pi}{3}\right)} \right]^2 \right\}^{0.5}$$

With further analytical calculation and reduction of similar:

$$\Delta U = \sqrt{\frac{4 \cdot \left(\frac{3 \cdot U_2 - 3 \cdot U_3}{2}\right)^2}{27} + \frac{4 \cdot \left(\frac{\sqrt{3} \cdot U_2 - \sqrt{3} \cdot U_1 + \frac{\sqrt{3} \cdot U_3}{2}\right)^2}{27}} = \frac{1}{3} \cdot \sqrt{(U_2 - 2U_1 + U_3)^2 + 3(U_2 - U_3)^2} = \frac{2}{3} \sqrt{U_1^2 - U_1U_2 - U_1U_3 + U_2^2 - U_2U_3 + U_3^2} = \frac{\sqrt{2}}{3} \sqrt{(U_1 - U_2)^2 + (U_1 - U_3)^2 + (U_2 - U_3)^2}$$

(7)

**APPENDIX C**

```

# -----
# Software for demodulation
# © Volodymyr Borovytsky, 2022
# -----
import math
import numpy as np
import cv2 as cv

# -----
# Function for demodulation
# using the known technique
# Modulation phases should be
# (0, 120, 240) ang. degrees
# -----
def dem_120 (au1, au2, au3):
    a1 = float(au1) - float(au2)
    a2 = float(au2) - float(au3)
    a3 = float(au1) - float(au3)

    sum = a1*a1 + a2*a2 + a3*a3
    coeff = math.sqrt(2.0)/3.0

    res= coeff* math.sqrt(sum)
    return res

# -----
# Function for demodulation
# using the proposed technique
# Modulation phases should be
# equal to (0, aph2, aph3)
# -----
def dem_func (au1, au2, au3, aph2, aph3):
    c32 = math.cos(aph3 - aph2)
    c3 = math.cos(aph3)
    c2 = math.cos(aph2)

    a1 = 2.0 * (1.0 - c32) * au1 * au1
    a2 = 2.0 * (1.0 - c3) * au2 * au2
    a3 = 2.0 * (1.0 - c2) * au3 * au3

    b1 = 2.0 * (c32 + c3 - c2 - 1.0) * au1 * au2
    b2 = 2.0 * (-c32 + c3 + c2 - 1.0) * au2 * au3
    b3 = 2.0 * (c32 - c3 + c2 - 1.0) * au1 * au3

    sum = a1 + a2 + a3 + b1 + b2 + b3
    dd = math.sin(aph2 - aph3) - math.sin(aph2) +
    math.sin(aph3)

    res = ( 1.0 * math.sqrt(sum) ) / abs( dd )
    return res

# -----
# This function performs demodulation
# of all pixels and it composes
# the output image
# -----
def my_dem( amod1, amod2, amod3, aph2, aph3,
aproposed):
    sx= amod1.shape[1]
    sy= amod1.shape[0]

    res = np.zeros( (sy, sx), dtype=np.uint8)

    rx=range(0,sx)
    ry=range(0,sy)

    for x in rx:
        for y in ry:
            u1 = amod1[y,x]
            u2 = amod2[y,x]
            u3 = amod3[y,x]

            if aproposed:
                u = dem_func( u1, u2, u3, aph2, aph3 )
            else:
                u = dem_120(u1, u2, u3)

            n = round( u )
            if n < 0:
                n = 0
            if n > 255:
                n = 255

            res[y,x] = n
    return res

# -----
# This function loads an image
# with spatial harmonic modulation

```

---

---

## МЕТОДИ ТА СИСТЕМИ ОПТИКО-ЕЛЕКТРОННОЇ І ЦИФРОВОЇ ОБРОБКИ ЗОБРАЖЕНЬ ТА СИГНАЛІВ

---

---

```
# -----
err = 0.0

def my_read_mod ( afn, anum):
    fn = afn.replace('0',str(anum+1))
    res = cv.imread( fn, cv.IMREAD_UNCHANGED )

    print("\n Modulation : num =', anum,' fn =', fn)
    print('  mod shape =', res.shape )
    print('  mod data type =', res.dtype )
    return res
# -----
# This function is the main program:
#
# 1. It loads 3 images with modulation
# 2. It performs demodulation
# 3. It saves the result of demodulation.
# -----

def main( afn ):
    mod1 = my_read_mod(afn, 0)
    mod2 = my_read_mod(afn, 1)
    mod3 = my_read_mod(afn, 2)

    phase_step = (2.0/3.0) * math.pi

    ph2 = phase_step - err
    ph3 = 2.0*phase_step + err

    # -----
    res = my_dem(mod1, mod2, mod3, ph2, ph3, True )
    # -----

    fn = afn.replace('0', 'D')
    print("\n Demodulation : fn =', fn)
    print('  mod shape =', res.shape )
    print('  mod data type =', res.dtype )

    cv.imwrite(fn, res)

    k = cv.waitKey()

    cv.destroyAllWindows()
    return k

main('c:\Photos\0.png')
```

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**БОРОВИЦЬКИЙ ВОЛОДИМИР МИКОЛАЙОВИЧ** – д.т.н., професор кафедри комп’ютерно-інтегрованих оптичних та навігаційних систем приладобудівного факультету НТУУ «КПІ ім. І.Сікорського», м. Київ, Україна, тел.: +380-44-204-94-77, [e-mail: vborovytsky@yahoo.com](mailto:vborovytsky@yahoo.com)

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### МЕТОДИ ТА ПРОГРАМНЕ ЗАБЕЗПЕЧЕННЯ ДЛЯ ЗЙОМКИ ТРИВИМІРНИХ ЦИФРОВИХ ЗОБРАЖЕНЬ ШЛЯХОМ ВИКОРИСТАННЯ ПРОСТОРОВОЇ ДЕМОДУЛЯЦІЇ

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