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ANALYSIS OF THE MAIN PROVISIONS OF THE THEORY OF PARALLEL-HIERARCHICAL TRANSFORMATIONS

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Анотація. У статті представлено аналіз основних положень теорії паралельно-ієрархічних перетворень. Постійний рух суспільства у сторону автоматизацій повсякденного життя вимагає створювати принципово нові програмні та апаратні рішення. Враховуючи поточні фізичні обмеження інтегральних схем, очевидним є шлях покращення програмної обробки. Головною проблемою цього є ріст складності архітектури та підтримки такого коду. Ідеї паралельно-ієрархічного мереж дозволяють значно збільшити швидкість обробки за рахунок паралелізації процесу зі збереженням відносної простоти архітектури програмного рішення. Запропонована структура ПІ-мережі дозволяє моделювати принцип роботи розподіленої нейронної мережі та за допомогою просторово-часового поділу утворює детерміновану мережу. Обговорено загальні правила прямого та зворотного паралельно-ієрархічного перетворення та його застосування для задачі розпізнавання образів. Показано блок-схему алгоритму для базової моделі нелінійного прямого мережевого перетворення. На прикладі використання представлена математична модель прямого паралельно-ієрархічного перетворення. Модель забезпечує складну функціональну обробку сигналів у масштабі реального часу, а також однозначність і оборотність з хорошою збіжністю обчислювального процесу.

Ключові слова: розпізнавання образів, паралельно-ієрархічна мережа, паралельно-ієрархічна трансформація

Abstract . The article presents an analysis of the main principles of parallel-hierarchical transformations theory. The continuous movement of society towards the automation of everyday life requires the creation of fundamentally new software and hardware solutions. Considering the current physical limitations of integrated circuits, it is evident that improving software processing is the way to go. The main problem lies in the increasing complexity of architecture and supporting such code. The ideas of parallel-hierarchical networks allow for a significant increase in processing speed through process parallelization while maintaining the relative simplicity of the software solution's architecture. The proposed structure of the parallel-hierarchical network allows for modelling the operation principle of a distributed neural network and forms a deterministic network using spatial-temporal division. The general rules of direct and inverse parallel-hierarchical transformation and their application to image recognition tasks are discussed. A block diagram of the algorithm for the basic model of nonlinear direct network transformation is shown. A mathematical model of direct parallel-hierarchical transformation is presented using an example. Compared to known numerical transformation methods involving simple operations like addition, the model enables complex functional signal processing in real-time scale, as well as unambiguity and reversibility with good convergence of the computational process.

Keywords: pattern recognition task, parallel-hierarchical network, parallel-hierarchical transformation

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INTRODUCTION

The growing volume of data and computations required to process large arrays of information, such as images, demands increasing performance from the systems used for this purpose.

Since the density of «packing» elements in integrated circuits is determined by physical limitations, the speed is ultimately limited by the propagation speed of electromagnetic oscillations from one element to another. This physical barrier can only be overcome by parallelizing computational processes in the system, which, in turn, leads to the complexity of its architecture. Intelligent information processing requires considering each element in a certain context of its connections, and this is only possible in a computational system that has a topographical structure with a three-dimensional placement of processor elements (PE).

ANALYSIS OF RECENT RESEARCH AND PROBLEM STATEMENT

Real objects and phenomena that are represented and studied using computational machines are rarely one-dimensional, and the processes of their transformation also rarely have a natural sequential nature. Therefore, numerous works in this field aim to bridge the gap between the real multidimensional and parallel world and the sequential approach to its representation in machine models. The obvious machine that can perform similar complex tasks is the brain. Therefore, it is not surprising that the work and theories of renowned neurobiologists like Mountcastle are used in the development of ideas and algorithms for image processing and pattern recognition [1-2]. The need to process large and massive arrays of information, such as images, in real time poses a problem for developers of digital devices: how to combine multidimensional data structures with structures for their transformation, storage, and processing. The mismatch between multidimensional data structures that form large input arrays and the structures for their transformation, processing, and storage in a one-dimensional computational environment leads, even in the case of a sequential processing process, to the need for significant computational work in data addressing. The unnatural and complex nature of computations and the methods of encoding digital information result in unnecessarily complex algorithmic and hardware solutions.

The purpose and tasks of the study. The unquestionable relevance of image recognition and analysis issues in the modern world presents complex tasks in creating solutions and mathematical models that can help automate the resolution of such problems using advanced technical capabilities while minimizing time and resource costs. The aim of the study is to analyze the developments in the theory of parallel-hierarchical transformations in order to find concepts that will aid in solving the aforementioned tasks and speeding up the operation of other image analysis models.

MATERIALS AND METHODS OF RESEARCH

Used in the theory of information structures and graph theory, the concepts of linear lists and trees are structures with ordered connections. In this sense, structural information (the connections between data elements) is predetermined by the type of structure itself and constitutes a structure with a flexible hierarchy. One way to achieve parallelism when working with complex interconnected structures is «regularization», which enables their description using a regular network transformation structure. In this case, information about the connections is explicitly included in the network transformation structure, i.e., it is represented by data elements. Let's focus on defining an abstract model of a network transformation structure.

Let's introduce some concepts related to the tree-like model of a network transformation structure with regular connections (*Figure 1*). Let graph $G = (V, E)$ be a data processing structure consisting of a set of nodes V and a set of edges E. The graph is directed if the edges are represented as ordered pairs of nodes. A transformation tree is defined by a directed graph that has the following properties: only root nodes have incoming edges, each node has a set of edges entering it, the number of which is determined by the nonlinear structure of the processed data, and there is a unique path from each root to a node, meaning a unique set of edges. The data processing structure is identified with a directed graph, where nodes correspond to data elements, and the directed edges connecting the nodes describe various dependencies between elements and are appropriately labelled.

In the process of hierarchical processing, the data structure changes from a large fixed array at the lower level to a small flexible structure at the upper level. Of particular interest are homogeneous non-distributed computational structures that correspond to the SIMD class of systems, where multiple levels of identical PEs work in SIMD mode. Each level contains a large number of simple PEs. The following systems belong to this group: PCLIP, PAPIA, GAM, SPHINX, as well as structures proposed by the author and protected by copyright certificate and a patent for the means of implementing hierarchical information processing. In a more complex case of homogeneous distributed hierarchical computational structures, several powerful identical processor blocks are combined into a hierarchical pyramid structure. Each processor block corresponds to a portion of the processed data. Such a pyramid system can operate in both SIMD and MIMD modes. This family includes computational structures proposed by the author and protected by copyright certificates for the means and device, as well as Uhrs Array/Net and EGPA systems.

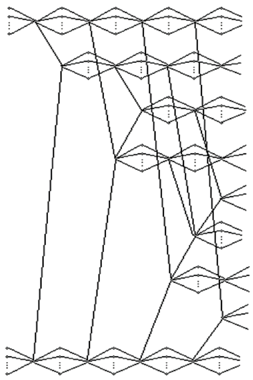

Figure 1 – A tree model of the network structure of the PI transformation

The principle of constructing a hierarchical pyramid data structure can be defined as a sequence of data arrays of the same information field at different resolution levels: $P=(A_0, A_1, A_2, \dots, A_l)$, where A_i is the information field, i is the resolution level, and $i=\overline{0,l}$. Such a pyramid of information fields forms a computational structure that enables the implementation of methods for intelligent sensor perception. In particular, this structure allows for controlling the resolution level of processed data and the size of the analysis area. The dimensions of the analyzed data «window» can be constant, but by moving from one placement level to another, processing of the same element of the information field can be performed with different levels of detail. In this case, the decision on the need for further processing can be made at the higher processing level after analyzing the information field with a low resolution, where each element contains integral assessments of the corresponding fragments in the original field at the lowest level, resulting in increased processing speed. The essence of the pyramid approach lies in the simultaneous use of a sequence of data arrays at different levels of hierarchy during analysis. This allows for implementing a strategy from «general to specific», enabling the realization of the concept of neural-like processing. Each element of the information field pyramid is characterized by three coordinates: (i,j,k), where i is the row, j is the column, and k is the level.

The principle of constructing a parallel-hierarchical data structure can be defined as a sequence of operations on sets of data arrays that form sets of information fields at different levels of hierarchy, with interaction between them realized through a pyramid hierarchical structure and implemented based on a network architecture. Network transformations are nonlinear transformations, whose kernels can be imagined as a network model. As a result of direct network parallel-hierarchical transformation of an image matrix $\mu(i,j)$ of size $S\times n$, a one-dimensional matrix $\varphi(t,p)$ will be formed, whose elements are defined as follows: if $\mu(i,j)=\sum_{i=1}^{n_s} a_i^j$ and number of levels $k=\sum_{p=0}^{c}(3p+2)$ where $c=0,1,2\dots$, then $\Phi^k_{t=2}\left[\sum_{j=1}^{S}\sum_{i=1}^{n_S}\mu(i,j)\right]=\varphi(t,p)$.

In this case, for each output element of the network transformation, the following relation holds true:

$$\varphi(t,p)=\Phi(i,j,t,p)\sum_{j=1}^{S}\sum_{i=1}^{n_S}\mu(i,j) \tag{1}$$

where $\Phi(i,j,t,p)$ - the kernel of the nonlinear direct network transformation defined over the elements of the input image. The relation (1) in vector form can be expressed as follows:

$$\varphi=\Phi^{\circ}f \tag{2}$$

where $f(i,j)=\sum_{j=1}^{S}\sum_{i=1}^{n_S}\mu(i,j)$.

The output image can be obtained through reverse network parallel-hierarchical transformation, which is described by the following relation:

$$f(j,i)=\Phi^{-}(i,j,t,p)\sum_{t=2}^{k}\sum_{p=0}^{c}\varphi(j,i) \tag{3}$$

where $\Phi^{-}(i,j,t,p)$ - nonlinear inverse network transformation kernel.

The inverse network transformation in vector form is given by:

$$f=\Phi^{-}\varphi \tag{4}$$

The flowchart of the algorithm for the core model of the nonlinear direct network transformation is shown in *Fig. 2*, from which it is straightforward to construct the flowchart of the algorithm for the model of the nonlinear inverse network transformation.

Currently, two approaches are competing in signal processing: the detector approach and the spatial-frequency approach. The detector approach assumes the existence of operators that extract the most frequently occurring elements of an image, allowing a transition from a point-wise description to larger image elements, significantly reducing redundancy. The spatial-frequency approach assumes that the visual cortex undergoes a transition to a quasi-holographic type of description, where the signal values are encoded not only at individual points but distributed in their vicinity. According to the spatial-frequency approach, cortical receptive fields describe images by decomposing them using a specified system of basic functions, such as the Fourier series. A highly economical spatial-frequency description is required for representation. Thus, for the recognition of a learned pattern, a highly degenerate description containing only a few (no more than five) harmonics is sufficient.

In simplified terms, the process of perceiving sensory signals from the human sensory organs can be represented as a interconnected iterative process of parallel set comparison and extraction of subsets of common and diverse (differential) signal attributes (SA). Each new set of differential SA generates a new subset of common SA through evolutionary change. This process of sensory information perception in the sensory organ systems occurs continuously over time and simultaneously (in parallel) in all systems of the human sensory organs. The set of common SA is directed to the cortex of the brain through independent pathways, creating a set of diverse common SA from which new sets of common and diverse SA are formed.

Each new set of common and diverse SA characterizes a new hierarchical level of sensor information processing. Samples from the external world are constructed in the later stages of sensory analysis by combining maximally purified (filtered) data that has passed through the filters of individual sensory systems.

In sensory systems, a hierarchical organization of information analysis and synthesis is characteristic. In some systems, an ascending nature of such filtration is provided, while in others, it is descending. The process of synthesis, which means the formation of mental images of the material world, occurs in reverse order, that is, from the preserved collection of various «representations» in the short-term (working) or long-term (permanent) memory of the brain, a new set of new and diverse «representations» is formed at each hierarchical level of the SA.

The merging of sensory data with hierarchically generalized information extracted from the chamber of human memory allows for a meaningful interpretation of collective general operating systems at each hierarchical level. Thus, at the structural level, the process of neuromorphic processing can be presented as a parallel-hierarchical and interconnected process of information analysis in a series of sensory zones with hierarchical levels of increasingly higher order, the highest of which form conglomerates of maximally general and diverse SA.

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In this plan, the proposed approach aligns with the fundamental concept of American physiologist V. Mountcastle. According to Mountcastle's theory, the cortex of the brain is composed of complex multicellular ensembles, with the primary unit consisting of approximately a hundred vertically connected neurons across all layers of the cortex, forming a larger unit called a module. Modules are grouped into larger assemblies, which form the primary visual, auditory, or motor cortex depending on where the particular assembly primarily receives information from. Information processing occurs in parallel channels, and each of these assemblies, performing a specific main function, also contains smaller subgroups of vertical units, each of which is associated with specific subgroups of other assemblies that primarily perform different functions. Interconnected subgroups are interconnected parts of a network that can extensively branch out throughout the cortex according to a defined law. The realization of the complex function, which is the ability to reach some form of abstract conclusion, is the result of the activity of the entire distributed network. Therefore, a network structure is proposed and investigated, which allows simulating the operation principle of a distributed neural network and, by means of spatial-temporal division, forms a deterministic network.

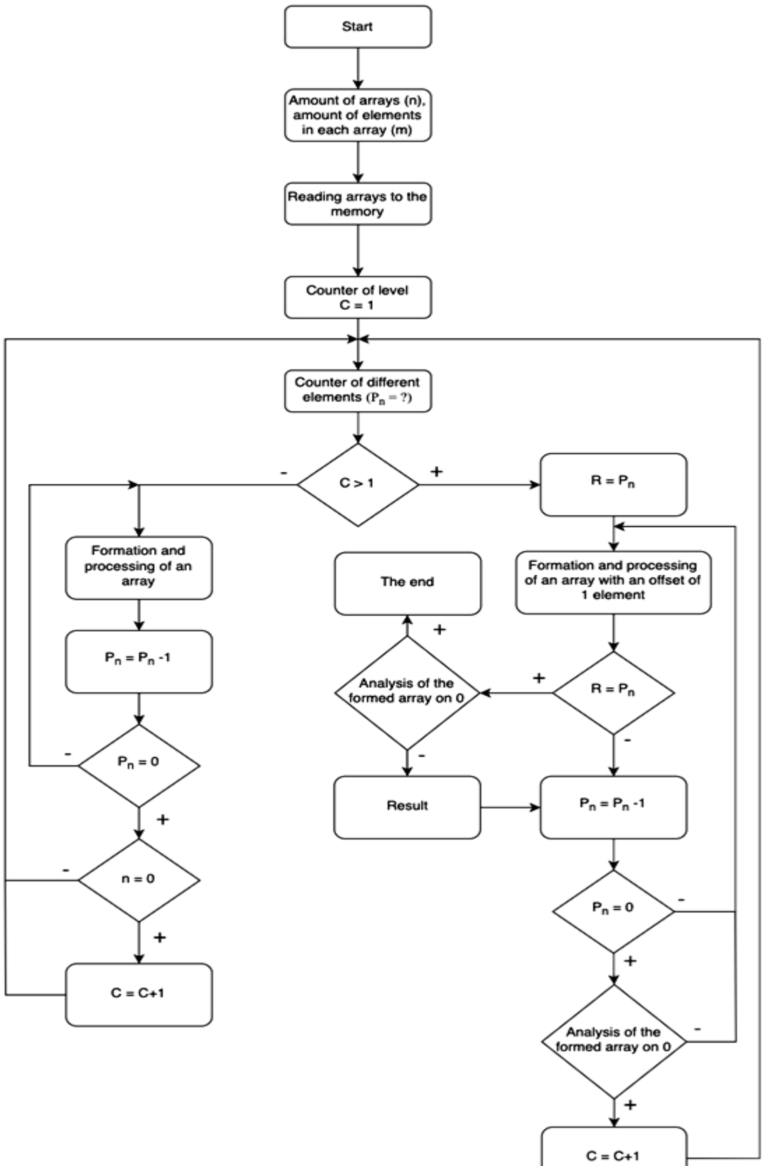

Figure 2. Graph scheme of the algorithm for the core model of the direct network PI transformation

The «intellectual» level of a distributed network is determined by the degree of generalization of sensory information in its branches. The higher the degree of generalization of sensory information as it passes through the network branches, the higher its «intellectual» level. The idea of using the formation of sets of general and diverse network states for processing sensory information coincides with the research of the well-known American psychologist D. Bruner, who experimentally proved that humans are prone to «seeing examples of a general rule in familiar individual cases». Based on his research, D. Bruner made an important assumption: the ability to form general rules from individual cases is part of the specific inheritance of human beings as a species. This assumption is also supported by the concept of «homeostasis», which emphasizes both «constancy» and the «relativity» of constancy, as outlined in the fundamental work of V. Kannon. According to Kannon, the constant conditions maintained within the body can be referred to as equilibrium or, in the proposed terminology, the general state of the system. The proposed and investigated network method combines the two competing approaches of nature's signal encoding: the detector-based approach and the spatial-frequency approach. It describes an image by decomposing it using an adaptive system of basis functions, the formation of which depends on the structure of the image.

The essence of the parallel-hierarchical approach lies in the simultaneous use of sequences of sets of data arrays that form sets of information fields at various levels of hierarchy, the recursive formation of new sequences of information streams at different levels of hierarchy, which allows implementing a multi-level interaction strategy from «general to specific». Each element of the parallel-hierarchical pyramid is characterized by four coordinates (i, j, k_1, k_2), where k_1 is the level of the first-level pyramid, k_2 is the level of the parallel-hierarchical pyramid of other levels. The pyramidal computational structure based on the parallel-hierarchical transformation will form a network in the form of a parallel-hierarchical pyramid. Here, each pyramid uses its own processing element (PE), and the number of PEs is determined by the total number of branches in the parallel-hierarchical network.

The structural model of the PI network organization can be described using six functional sets, which allow formalizing the structural organization of spatial connections in the PI network. To analyze the structural-functional organization of the PI network, we introduce the following concepts and definitions.

Definition 1. The convolution of elements with coordinates $a_{1i}(t_j)$and $a_{2i}(t_j)$ from the p_{1j}-th and $p_{1(j+1)}$-th branches of the j-th level consists of elements with coordinates $a_{(i+1)1}(t_{j+1})$ and $a_{(i+1)2}(t_{j+2})$ from the p_{1j}-th branch of the $(j+1)$-th level. The truncated convolution of elements with coordinates $a_{1i}(t_j)$ and $a_{2i}(t_j)$ from the p_{1j}-th and $p_{1(j+1)}$-th branches of the j-th level is an element with coordinates $a_{(i+1)1}(t_{j+1})$ from the p_{1j}-th branch of the $(j+1)$-th level.

Definition 2. The unfolding of elements with coordinates $a_{(i+1)1}(t_{j+1})$ and $a_{(i+1)2}(t_{j+2})$ from the p_{1j}-th branch of the $(j+1)$-th level consists of elements with coordinates $a_{1i}(t_j)$ and $a_{2i}(t_j)$ from the p_{1j}-th and $p_{1(j+1)}$-th branches of the j-th level. The truncated unfolding of an element with coordinate $a_{(i+1)1}(t_{j+1})$ the p_{1j}-th branch of the $(j+1)$-th level consists of elements with coordinates $a_{1i}(t_j)$ and $a_{2i}(t_j)$ from the p_{1j}-th and $p_{1(j+1)}$-th branches of the j-th level.

Similarly, to Definitions 1 and Definitions 2, the concepts of folding, unfolding, and truncated unfolding can be introduced for multiple variables.

Definition 3. The current convolution of elements $a_1$and a_2 is the data folding of the elements at any given moment in time t_j

Definition 4. The previous convolution of network elements with respect to the current convolution is a convolution of the network that lags by the same number of convolutions from the tail element with respect to the current convolution.

For example, when encoding the current convolution PI transformation, it is necessary to analyze the mask function of the previous convolution. If the mask function of the given convolution is equal to one, then the current convolution consists of a single branch element, whose content is transferred to the neighboring convolution element according to Definition 2, and the truncated convolution operation is implemented.

To demonstrate the direct and reverse PI transformation, let's introduce the concept of a subnetted network $C^{(m)}(i, j)$, which is formed by the main network $C_1^{(m)}(i, j)$, and the subnetted networks $C_2^{(m)}(i, j)$. In a complete basis network, more than 50% of the convolutions at $n \geq 4$are composed of two elements. This regularity is significant, for example, in the development of neural networks for data compression. By utilizing the truncated convolution operation and its masking function, as well as the structural-functional model of the basis network, it is possible to represent the input information stream in a compact (compressed) form. Encoding based on PI transformation is implemented using the following general rules.

1. When encoding the current convolution of a PI network, an analysis of the mask function of the previous convolution takes place. If the given function is $C^{(m)}(i,j)=1$, the mask function of the current convolution is not stored. Otherwise, when it is $C^{(m)}(i,j)=0$, the mask function of the current convolution is fixed. In this case, if it is $C^{(m)}(i,j)=1$, the content of an element from the previous (lower) level of convolution is passed to an element in the next (higher) level with the corresponding address, whose content is non-zero. In the other case, if the mask function of the previous convolution is $C^{(m)}(i,j)=0$, the content of an element from the previous (lower) level is passed to an element in the next (higher) level with the same address.
2. The memorization of the content of the tail elements of the network is taking place.
3. Content routing of incoming elements occurs from one level (lower) to another level (higher) towards its tail element.
4. If the mask function of the previous convolution is, then the current convolution is formed from a single element that will create a truncated convolution at the next level. In the opposite case, when, the current convolution is formed from two elements that create a convolution at the next level of the network.
5. If the previous convolution is formed from a single element, i.e., a truncated convolution, then the mask function of the current convolution is not stored.
6. If the elements of current convolutions correlate (match) with one of the variants of the elements of previous convolutions, the mask functions of which are $C^{(m)}(i,j)=0$, then the mask functions of such current convolutions are also not memorized.

The general rules for implementing decoding are as follows.

1. Decoding is performed from the content of each tail element of the network, starting from the last one.
2. Routing of the content of the tail element occurs from the previous (smaller) level to the next (larger) level.
3. If the masking function of the previous convolution of the elements, which are formed when unfolding the next convolution element, is equal to one, i.e., the content of these same elements matches in magnitude during encoding, then the content of the previous-level element is transmitted to the next-level element, whose content is nonzero. Otherwise, the masking function of the previous convolution elements is equal to zero. In other words, if the content of these elements does not match in magnitude during encoding, then the content of the previous-level element is transmitted to the next-level element with the same address.

The mentioned general transformations for PI encoding and decoding rules are applied in algorithms of parallel-hierarchical computational structures and in tasks of compressing data information streams, discussed in the fourth chapter of the work. To study the general regularities of the structural-functional organization of the PI network, we will present a diagram illustrating the formation of various convolutions from two dominant elements in the network, and a diagram illustrating the formation of tail elements (*Figure 3*) of the PI network.

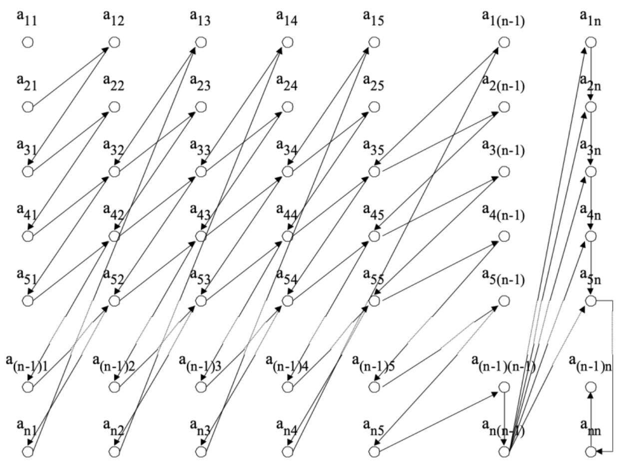

Figure 3 – Scheme for generating addresses for various combinations of two elements in a network for the dimensional data flow $n \times n$

When analyzing the current dual convolutions of the network, if the masking function precedes the dual convolution $C^{(m)}(i, j) = 1$, a situation arises where additional dual convolutions are formed. The possible options for forming such convolutions for the input dimension 4×4 are shown in *Fig. 4*. The total number of diverse dual convolutions (excluding additional convolutions) N in the data PI network is determined by:

$$N = N^2 \tag{5}$$

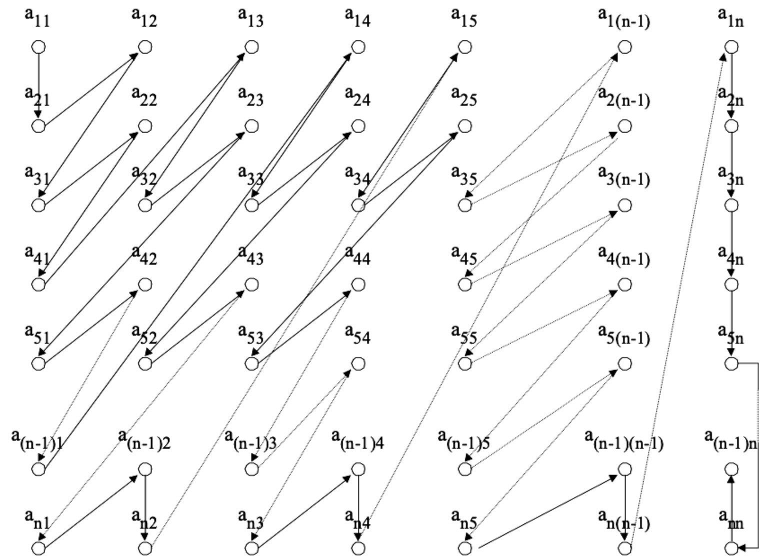

Figure 4. The block diagram for generating tail element addresses in a PI network for the dimensionality of data flow $\boldsymbol{n} \times \boldsymbol{n}$

In turn, the formation of additional double convolutions (*Fig. 5*) with a mask function $C^{(m)}(i, j) = 1$ leads to the formation of truncated convolutions, resulting in opportunities to enhance the efficiency of PI transformation.

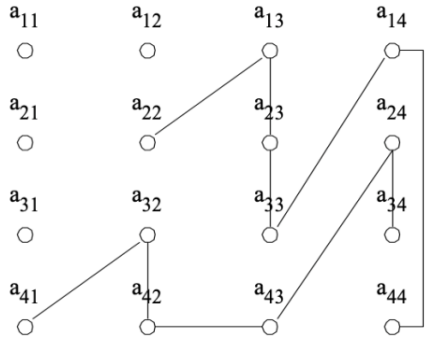

Figure 5. Possible options for creating additional dual convolutional folds of network elements (for the input dimension $\boldsymbol{n} \times \boldsymbol{n}$)

Let's formulate the mathematical model of direct parallel hierarchical transformation. Consider the network method of PI transformation at the model level. Let there be S non-empty sets of elements that define information. The number of elements in a set is called its length (denoted by L_{μ} - the length of set μ). The number of distinct elements in a set is called the dimensionality of that set (denoted by R_{μ}). Let's consider the mathematical model of parallel decomposition of set $\mu = \{a_i\}$, $i = \overline{1,n}$:

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{R} \left(n - \sum_{k=1}^{j-1} n_k \right) \left(a^j - a^{j-1} \right) \tag{6}$$

where $a_i \neq 0$, R- is the cardinality of the given set. From the identical elements, we will form subsets. The elements of each subset will be denoted by a^k, $k=\overline{1,R}$, n_k - the number of elements in the k-th subset (i.e., the multiplicity of the number a_k), and a^j - an arbitrary element of the set $\{a^k\}$ chosen at the j-th step, $j=\overline{1,R}$, $a^0=0$, $n_0=0$.

If we replace the symbol $\sum$ with the symbol $\bigcup$ in equation (6) and perform the operation, the result of decomposing the set μ is the union of its elements (the set of decomposition μ^1):

$$\mu=\bigcup_{i=1}^{n} a_i=\bigcup_{j=1}^{k}\left(n-\sum_{k=1}^{j-1} n_k\right)\left(a^j-a^{j-1}\right). \quad (7)$$

The transformation of set μ into set μ^1, which is defined by model (7), will be called the transformation operator G - a type of Q^*-transformation that is:

$$G(\mu)=\mu^1. \quad (8)$$

If we apply the transformation operator G to the input arrays S, which is defined by formula (6), then for each array we will obtain its respective sequential decomposition. Let's combine the obtained elements into a matrix M_1. We will call it the matrix of direct decomposition at the first level. Then it will have the following form:

$$M_1=\bigcup_{S=1}^{S}\left(\bigcup_{i=1}^{R_S^1} a_{si}^1\right) \quad (9)$$

We will rewrite matrix A by grouping its elements not by rows but by columns, and in this way, we will form a new matrix:

$$M_1^T=\bigcup_{S=1}^{S}\left(\bigcup_{i=1}^{R_S^1} a_{si}^1\right)=\bigcup_{i=1}^{R_S^1}\left(\bigcup_{i=1}^{S} a_{si}^1\right) \quad (10)$$

Let's denote by T the matrix transposition operator, which corresponds to the transition from (9) to (10), then $M_1^T=T(M_1)$. Therefore, if R_S^1 takes on various values for each set, the number of sets in μ_1^T will be equal to the maximum of the numbers R_S^1, which is $\max\left\{R_1^1, R_2^1, R_3^1 ... R_S^1\right\}=R_1$. Thus, we denote the maximum length of a set among all sets in the decomposition at the first level as R_1. The matrix M_1^T consists, accordingly, of sets μ_S^{1T}, which transition to the next level and serve as the basis for constructing matrix M_2. Sequentially applying operator G to sets μ_S^{1T}, we will form a new array μ_2 at the second level $(k=2)$:

$$\mu_2=\bigcup_{R^1=1}^{R^1}\mu_{R^1}^2=\bigcup_{R^1=1}^{R^1}\left(\bigcup_{i=1}^{R_{R^1}^2} a_{R_{1_i}}^2\right) \quad (11)$$

Starting from the second level, the formation of matrices $M_1^T, M_2^T, M_3^T, ..., M_k^T$ occurs not only through transposition. We will group the elements not by columns but by diagonals, and the resulting sets will be called diagonal sets.

From the above, the problem arises as to how many levels are ensured for direct PI transformation. The sequential application of three operators G, S, T is called functional Φ, that is $\Phi_{t=2}^k[T(S(G(M)))]$ The following model is fair:

$$\Phi_{t=2}^k\left[T\left(G\left(\bigcup_{S=1}^{S}\mu_S\right)\right)\right]=\bigcup_{t=2}^{k} a_{11}^t, \quad (11)$$

μ_S – initial sets ($S = 1,2,3...$), a_{11}^t - the elements of decomposition of initial sets obtained one by one at each level, starting from the second.

$$\Phi_{t=2}^k\left[T\left(G\left(\bigcup_{S=1}^{S+1}\mu_S\right)\right)\right]=\left[\bigcup_{t=2}^k a_{11}^t\right]\cup\left[\bigcup_{j=1}^{R_{\mu_{S+1}}}\left(n_{\mu_{S+1}}+\sum_{k=0}^{j=1}n_{k_{\mu_{S+1}}}\right)\right] \tag{12}$$

Thus, the network method of direct PI transformation involves the sequential application of transformation operators G and transpose operator T to the initial sets $\bigcup_{S=1}^{S}\mu_S$, followed by $(k-1)$ iterations of the functional Φ. At each level of the PI transformation, one element of the decomposition S of output sets a_{11}^k is formed, where a_{11}^k represents the output information of the direct PI transformation. This information consists of diagonal sets with a common diagonal multiple element.

Result 1. The maximum number of hierarchical levels is one unit greater than the overall dimensionality of all initial sets.

Result 2. The sum of input elements of the network is equal to the sum of its tail elements.

To demonstrate the species model (11), let's consider a numerical example of direct PI transformation using, for example, the G transformation, with information given in the form of numerical sets μ_1, μ_2, μ_3:

$$\mu_1=\begin{pmatrix}2\\3\\5\\7\end{pmatrix};\ \mu_2=\begin{pmatrix}1\\4\\9\\10\end{pmatrix};\ \mu_3=\begin{pmatrix}3\\15\\20\end{pmatrix} \tag{13}$$

Using equation (6), we obtain:

$$G(\mu_1)=\begin{pmatrix}8 & 3 & 4 & 2\end{pmatrix};\ G(\mu_2)=\begin{pmatrix}4 & 9 & 10 & 1\end{pmatrix};\ G(\mu_3)=\begin{pmatrix}9 & 24 & 5\end{pmatrix} \tag{14}$$

From the obtained results, we will construct matrices:

$$M_1=\begin{pmatrix}8 & 3 & 4 & 2\\4 & 9 & 10 & 1\\9 & 24 & 5 & X\end{pmatrix};\ M_2=\begin{pmatrix}12 & 8 & 1 & X & X\\X & 9 & 12 & 15 & X\\X & X & 12 & 2 & 5\\X & X & X & 2 & 1\end{pmatrix}\Rightarrow a_{11}^1=12;$$

$$M_3=\begin{pmatrix}16 & 1 & X & X & X\\X & 3 & 22 & X & X\\X & X & 6 & 13 & X\\X & X & X & 2 & 4\end{pmatrix}\Rightarrow a_{11}^2=16;\ M_4=\begin{pmatrix}2 & 2 & X & X\\X & 12 & 16 & X\\X & X & 4 & 11\\X & X & X & 4\end{pmatrix}\Rightarrow a_{11}^3=2;$$

$$M_5=\begin{pmatrix}4 & 10 & X & X\\X & 8 & 12 & X\\X & X & 8 & 7\end{pmatrix}\Rightarrow a_{11}^4=4;\ M_6=\begin{pmatrix}16 & 2 & X\\X & 16 & 4\\X & X & 7\end{pmatrix}\Rightarrow a_{11}^5=16;$$

$$M_7=\begin{pmatrix}4 & 14 & X\\X & 8 & 3\end{pmatrix}\Rightarrow a_{11}^6=4;\ M_8=\begin{pmatrix}16 & 6\\X & 3\end{pmatrix}\Rightarrow a_{11}^7=16;$$

$$M_9=\begin{pmatrix}6 & 3\end{pmatrix}\Rightarrow a_{11}^8=6;\ M_{10}=(10)\Rightarrow a_{11}^9=3.$$

The result of the direct PI transformation is:

$$\Phi_{t=2}^{10}\left[T\left(G\left(\bigcup_{i=1}^{3}\mu_i\right)\right)\right]=(12,\ 16,\ 2,\ 4,\ 16,\ 4,\ 16,\ 6,\ 3);\ \sum_{i=1}^{3}\mu_i=79. \quad (15)$$

For this numerical example, the main property of the PI network used for its training procedure is:

$$\sum_{i=1}^{3}\mu_i=\sum_{i=1}^{10}a_i=12+16+2+4+16+4+16+6+3=79. \quad (16)$$

It is interesting to note that at the current level t of network processing, the sum of tail elements at the $(t-1)$-th and $(t-2)$-th levels, plus the sum of all elements except the tail element of the t-th level, is equal to the sum of all tail elements of k levels - $a_{11}^{t-1}-a_{11}^{t-2}+\sum_{i=2}^{N}a_i^t=\sum_{i=t}^{k}a_{11}^t$, where N is count of elements for t-level. This property of network computations is related to one of the key properties of Fibonacci numbers. From the considered statements and example, it is evident that the model of type (11) fully describes the network PI transformation.

CONCLUSIONS

The current research aimed to analyze the main provisions of the theory of parallel-hierarchical transformations in order to find concepts that will help reduce time and resource costs when solving image recognition problems. General rules for direct and inverse PI transformation of data have been formulated. A mathematical model of direct parallel hierarchical transformation is presented, and its application is demonstrated using an example. The proposed PI transformation approach assumes such a method of information processing, the implementation of which combines dynamic hierarchy and parallelism with a natural network transformation of multidimensional data structures. The model, in comparison with known numerical methods of transformation (for example, expansion into mathematical series) by simple addition-type operations, provides complex functional processing of signals on a real-time scale, as well as unambiguousness and reversibility with good convergence of the computational process. Moreover, the network method of processing leads to the rapid compression of input arrays of information. Fields of application of the network method of PI transformation are diverse. These are network structures: parallel memory, digital information reception-transmission systems, digital information compression devices, multi-gradation image comparison systems, including correlation comparison, digital image pre-processing devices, segmentation, coding, formation of features for recognition, and biomedical information processing.

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ОСОБЛИВОСТІ ВИКОРИСТАННЯ ТЕОРІЇ ПАРАЛЕЛЬНО-ІЄРАРХІЧНОГО ПЕРЕТВОРЕННЯ ДЛЯ ОБРОБКИ ІНФОРМАЦІЇ

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